Geometric Modelling of the Linear Delta

Introduction

The Robot « linear Delta »* is a variant of the robot DELTA invented by Professor Reymond Clavel in 1985. It is a 3 DOFs translational parallel structure providing the X, Y and Z movements of the mobile plate. The actuated movements are all linear guided and this is why this variant is called a « **linear Delta** »*

Descriptions and parametrization

The figure below represents different linear Delta variants.

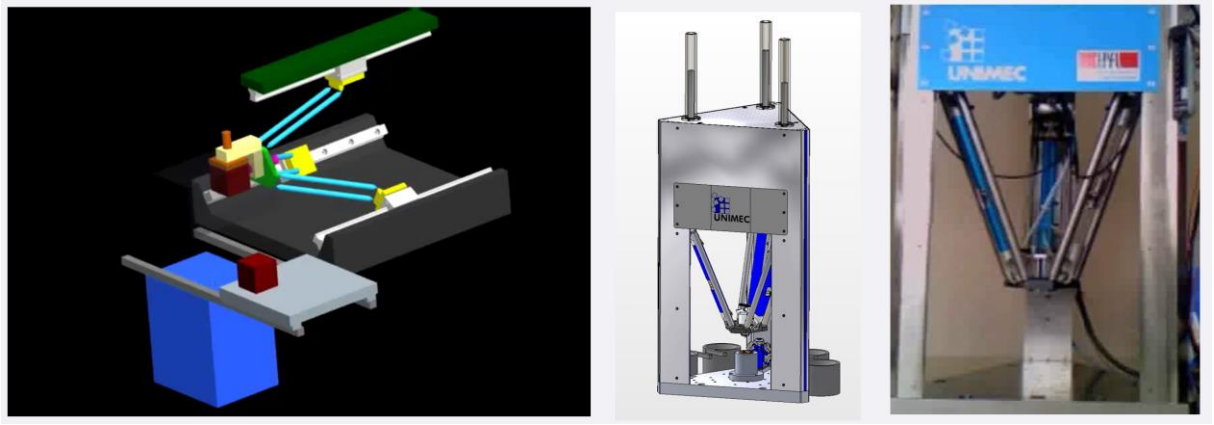

Figure 1. Different Linear Delta realizations

All of these structures are realized with a ternary symmetry which means that, from a top view perspective, the linear guideways belong to a circle with 120° between each point (fig. 2).

Parameters:

- The length of the parallel bars is **Lb**.
- The diameter of the base is 2x**Ra**
- The diameter of the mobile plate (nacelle) is 2x**Rb**
- **q1**, **q2** and **q3** are the joint coordinates, ie. the value of the positions of the **actuated carriages** of each linear guideway. The points A1, A2 and A3 are associated to the actuated carriages

- The vertical axis **Z** is referenced at the center and the top of the robot (as in the figure). X and Y are shown in the figures 3 and 4.

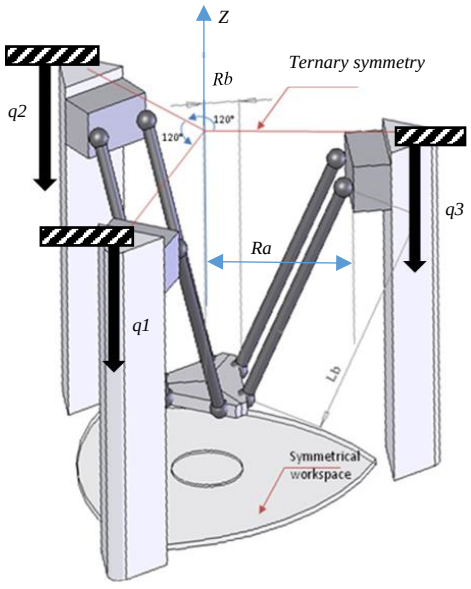

Figure 2. Parametric representation of the linear Delta

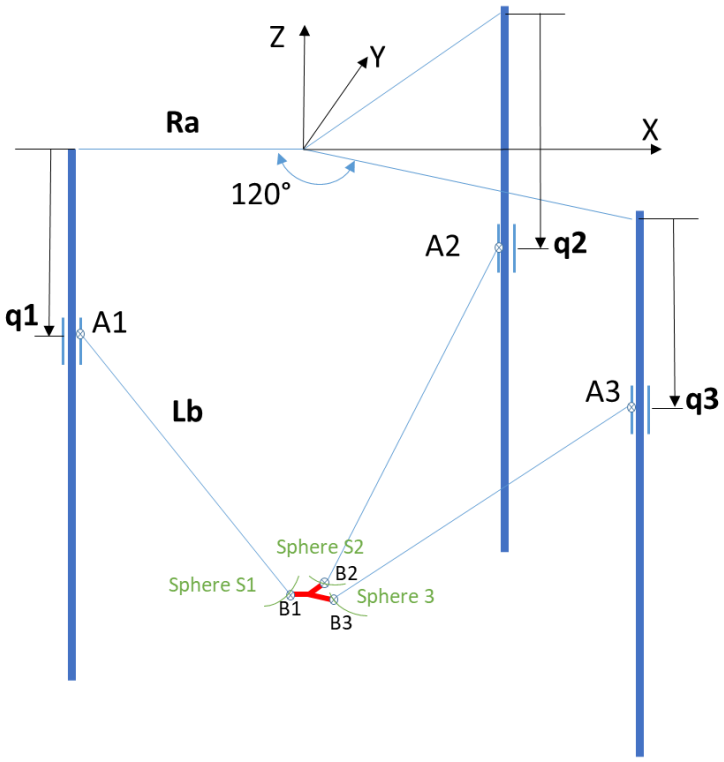

Figure 3. Parametric representation of the linear Delta.

The parallel bars are simplified with one segment each of length Lb. The double spherical joints at each extremity of each parallel bar are represented by a universal joint. The movement of each universal joint, respectively at A1, A2 or A3, generate a spherical calotte of diameter Lb and respectively cross the mobile plate points B1, B2 and B3.

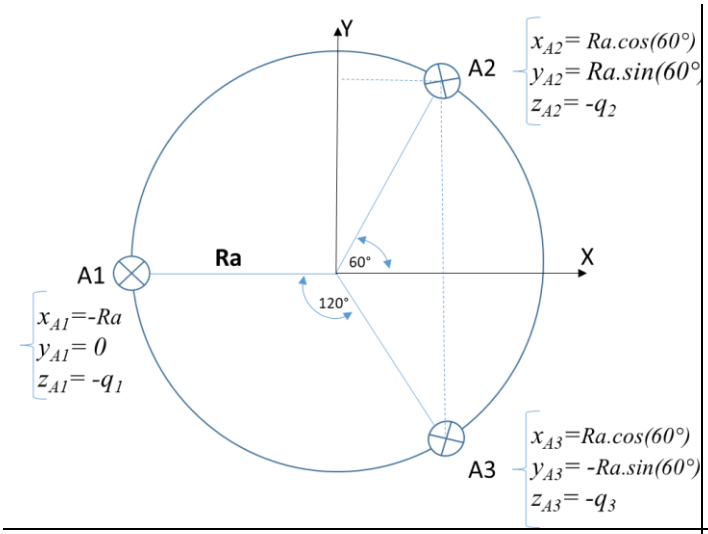

Figure 4.a, top-view of the linear Delta. The actuated points A1, A2 and A3 are the represented by universal joints.

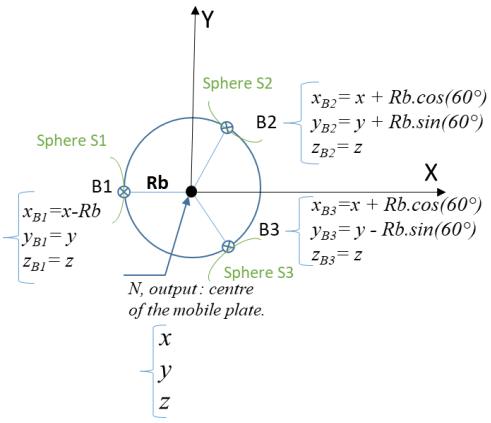

Figure 4.b, top-view of the mobile plate. The points B1, B2 and B3 are the represented by universal joints.

Movement of the end effector and equations

As described in the figure 3, *the parallel bars are simplified with one segment each of length Lb. The double spherical joints at each extremity of each parallel bar are represented by a universal joint. The movement of each universal joint, respectively at A1, A2 or A3, generate a spherical calotte of diameter Lb and respectively cross the mobile plate points B1, B2 and B3.* This notice is at the basis of the equations of the movement of the mobile plate. The 3 spheres **S1** and **S2** and **S3** are respectively centred at the points **A1** and **A2** and **A3** and cross the mobile plate respectively at the points **B1** and **B2** and **B3**. The coordinates of all these points are given the figures 4.a and 4.b.

The objective:

The goal of geometric modelling, is to find the coordinate transformation between the joint coordinates (actuated movements) {***q1, q2, q3***} and the coordinates of the end effector {***x, y, z***}.

The equations of the three spheres may be described as follows:

{S1} $(x_{B1} - x_{A1})^2 + (y_{B1} - y_{A1})^2 + (z_{B1} - z_{A1})^2 = Lb^2$

{S2} $(x_{B2} - x_{A2})^2 + (y_{B2} - y_{A2})^2 + (z_{B2} - z_{A2})^2 = Lb^2$

{S3} $(x_{B3} - x_{A3})^2 + (y_{B3} - y_{A3})^2 + (z_{B3} - z_{A3})^2 = Lb^2$

Let us focus on the equation describing the sphere S1, by replacing the values of each coordinate.

{S1} $(x_{B1} - x_{A1})^2 + (y_{B1} - y_{A1})^2 + (z_{B1} - z_{A1})^2 = Lb^2$

$(x - Rb + Ra)^2 + (y)^2 + (z + q_1)^2 = Lb^2$

$(z + q_1)^2 = Lb^2 - (x - Rb + Ra)^2 - (y)^2$

$(z + q_1) = \pm\sqrt{Lb^2 - (x - Rb + Ra)^2 - (y)^2}$ (2 solutions are possibles)

$z = -q_1 \pm \sqrt{Lb^2 - (x - Rb + Ra)^2 - (y)^2}$

At this stage, 2 solutions are possible. In such case we always should understand how the robot behave in order to try finding impossible situations thanks to the information of limits, sign and geometrical boundaries. We can discriminate these 2 solutions by discarding the one which gives a possible positive z.

Hence, since the z coordinate is always negative, the final solution is given by the following equations:

$$z = -q_1 - \sqrt{Lb^2 - (x - Rb + Ra)^2 - (y)^2}$$

$$q_1 = -z - \sqrt{Lb^2 - (x - Rb + Ra)^2 - (y)^2}$$

The equation of the sphere S2 is given as follows:

{S2} $(x_{B2} - x_{A2})^2 + (y_{B2} - y_{A2})^2 + (z_{B2} - z_{A2})^2 = Lb^2$

$(x + Rb.\cos(60°) - Ra.\cos(60°))^2 + (y - Rb.\sin(60°) - Ra.\sin(60°))^2 + (z + q_2)^2 = Lb^2$

$(x - (Ra - Rb).\cos(60°))^2 + (y - (Ra - Rb).\sin(60°))^2 + (z + q_2)^2 = Lb^2$

$(z + q_2)^2 = Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y - (Ra - Rb).\sin(60°))^2$

$(z + q_2) = \pm\sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y - (Ra - Rb).\sin(60°))^2}$

$q_2 = -z \pm \sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y - (Ra - Rb).\sin(60°))^2}$

For the same reasons as for q_1, since z is always negative, the adopted expression of q_2 is as follows:

$$q_2 = -z - \sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y - (Ra - Rb).\sin(60°))^2}$$

The equation of the sphere S3 is given as follows:

{S3} $(x_{B3} - x_{A3})^2 + (y_{B3} - y_{A3})^2 + (z_{B3} - z_{A3})^2 = Lb^2$

$(x + Rb.\cos(60°) - Ra.\cos(60°))^2 + (y - Rb.\sin(60°) + Ra.\sin(60°))^2 + (z + q_3)^2 = Lb^2$

$(x - (Ra - Rb).\cos(60°))^2 + (y + (Ra - Rb).\sin(60°))^2 + (z + q_3)^2 = Lb^2$

$(z + q_3)^2 = Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y + (Ra - Rb).\sin(60°))^2$

$(z + q_3) = \pm\sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y + (Ra - Rb).\sin(60°))^2}$

$q_3 = -z \pm \sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y + (Ra - Rb).\sin(60°))^2}$

Since z is always negative, the expression of q_3 is as follows:

$$q_3 = -z - \sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y + (Ra - Rb).\sin(60°))^2}$$

The inverse geometric model is finally given by :

$$\boldsymbol{q_1 = -z - \sqrt{Lb^2 - (x - Rb + Ra)^2 - (y)^2}}$$

$$\boldsymbol{q_2 = -z - \sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y - (Ra - Rb).\sin(60°))^2}}$$

$$\boldsymbol{q_3 = -z - \sqrt{Lb^2 - (x - (Ra - Rb).\cos(60°))^2 + (y + (Ra - Rb).\sin(60°))^2}}$$

Observation:

In the case of this robot with a ternary symmetry, another procedure of obtaining the geometric model may be carried out by reducing the size of the basis by Rb (ray of the mobile place). This does not modify anything to the output. The mobile plate is hence reduced to a one point of coordinates {x, y, z}. The figures below point out the procedure:

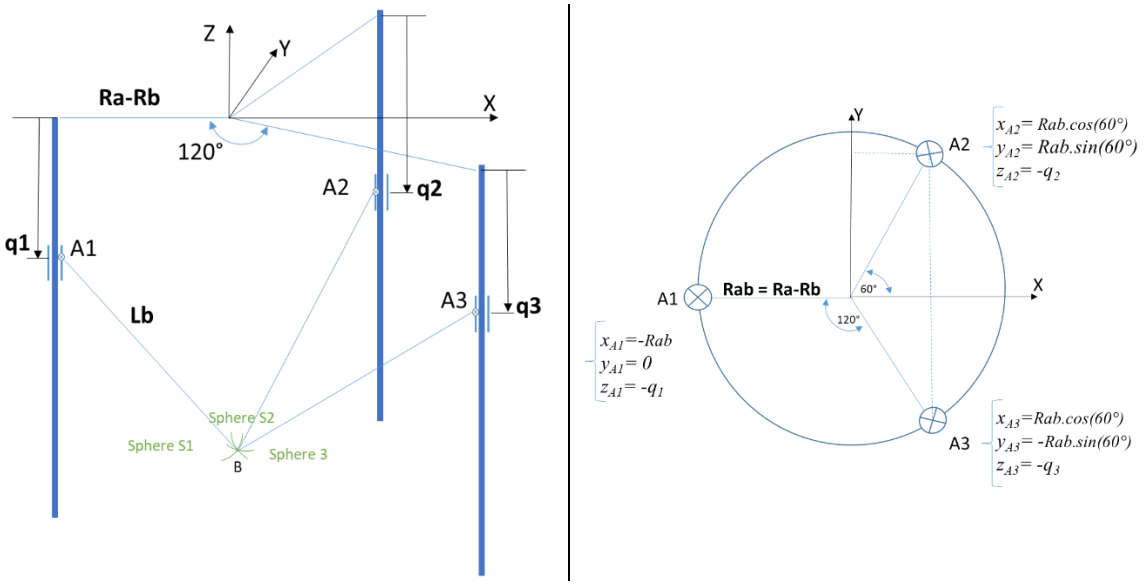

Figure 5. Parametric representation of the linear Delta by bringing the parallel bars to the centre of the mobile plate (by reducing the basis by Rb). (Left) perspective. (Right) top view. *The movement of each universal joint, respectively at A1, A2 or A3, generate a spherical calotte of diameter Lb and respectively cross the mobile plate at the centre of the mobile plate B.*

The three spheres intersect all at the tool centre point and are given by the following equations:

{S1} $(x - x_{A1})^2 + (y - y_{A1})^2 + (z - z_{A1})^2 = Lb^2$

{S2} $(x - x_{A2})^2 + (y - y_{A2})^2 + (z - z_{A2})^2 = Lb^2$

{S3} $(x - x_{A3})^2 + (y - y_{A3})^2 + (z - z_{A3})^2 = Lb^2$

Hence given by:

{S1} $(x - Rab)^2 + (y)^2 + (z + q_1)^2 = Lb^2$

{S2} $(x - Rab.\cos(60°))^2 + (y - Rab.\sin(60°))^2 + (z + q_2)^2 = Lb^2$

{S3} $(x - Rab.\cos(60°))^2 + (y + Rab.\sin(60°))^2 + (z + q_3)^2 = Lb^2$

With Rab = Ra-Rb.

Which correspond to the same equations as before and avoid expressing the coordinates of the intersecting points (B1 and B2 and B3).at the mobile plate